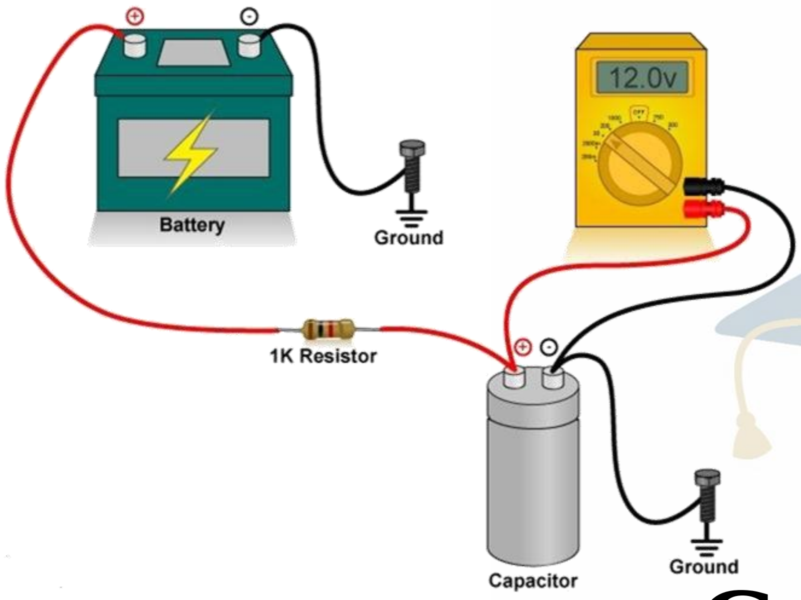


Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

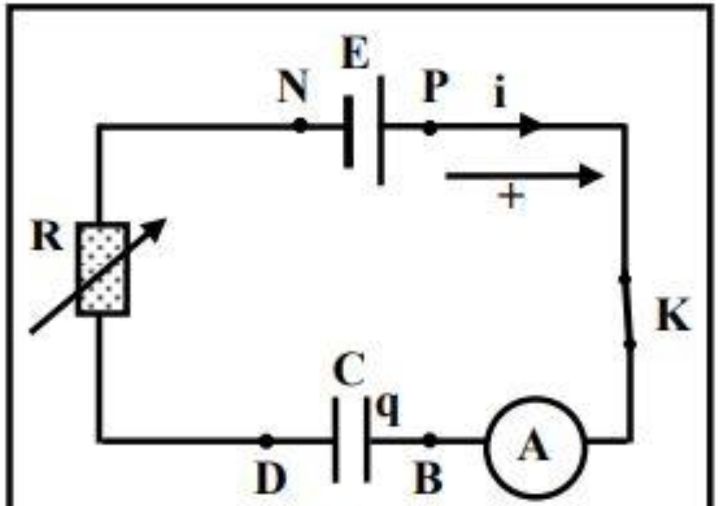
Prepared & Presented by: Mr. Mohamad Seif

Think then Solve

Exercise 1:

To determine the capacitance C of a capacitor We set-up the series circuit of (Doc 4).

This circuit includes: an ideal battery of electromotive force $E = 10\,V$; a rheostat of resistance R; a capacitor of capacitance C; an ammeter and a switch K.



Doc.4

Initially the capacitor is uncharged. We close the switch K at the instant $t_0 = 0$.
At an instant t, plate B of the capacitor carries a charge q and the circuit carries a current i as shown in doc 4.

Exercise 1:

1. Write the expression of i in terms of C and u_C, where $u_C = u_{BD}$ is the voltage across the capacitor.

2. Establish the differential equation that governs the variation of u_C.

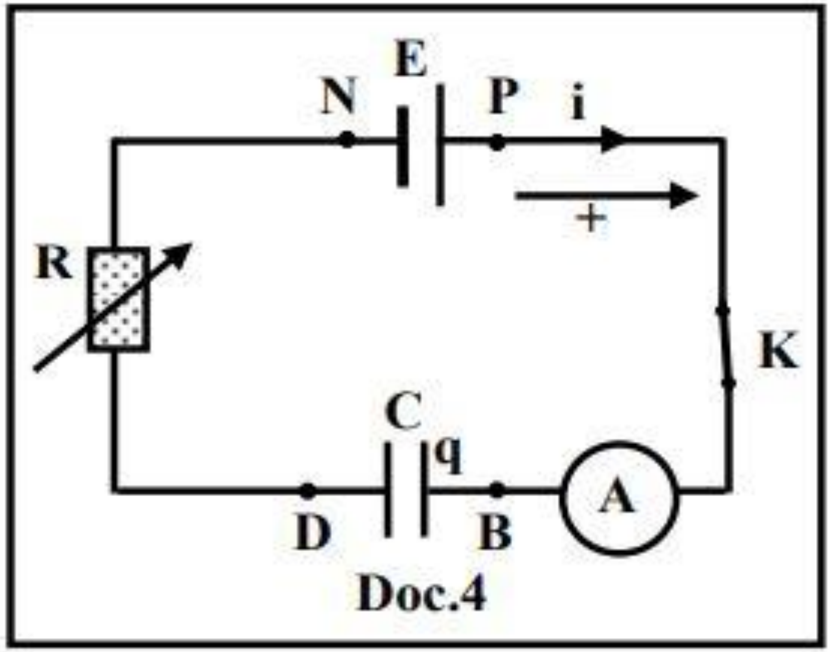
3. The solution of the differential equation is: $u_C = a + b.e^{-\frac{t}{\tau}}$. Determine the expressions of the constants a, b and τ in terms of E, R and C.

4. Deduce that the expression of the current is: $i = \frac{E}{R}.e^{-\frac{t}{RC}}$.

5. The ammeter indicates $I_0 = 5mA$ at $t_0 = 0$. Deduce the value of R.

Exercise 1:

1. Write the expression of i in terms of C and u_C, where $u_C = u_{BD}$ is the voltage across the capacitor.



Doc.4

$$i = \frac{dq}{dt}$$

But $q = C.u_c$

$$i = \frac{dC.u_C}{dt}$$

$$i = C\frac{du_C}{dt}$$

Exercise 1:

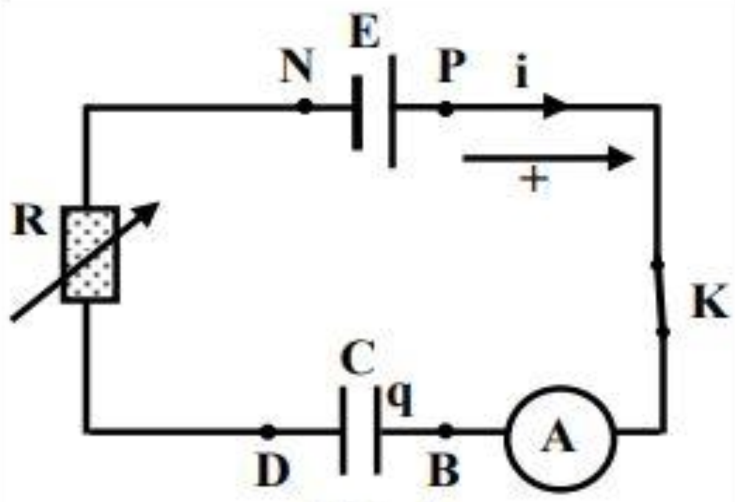
2. Establish the differential equation that governs the variation of u_C.

Using law of addition of voltages in series:

$$u_G = u_C + u_R$$

$$\mathrm{E} = \mathrm{u}_C + \mathrm{Ri}$$

$$\mathrm{E} = u_C + \mathrm{RC}\frac{du_C}{dt}$$



Doc.4

Exercise 1:

3.The solution of this differential equation is $u_C = a + b.e^{-\frac{t}{\tau}}$. Determine the expressions of the constants a, b and τ in terms of E, R and C.

$$u_C = \mathrm{a} + be^{-\frac{t}{\tau}}$$

$$\frac{du_C}{dt} = -\frac{b}{\tau}e^{-\frac{t}{\tau}}$$

$$\mathrm{E} = u_C + RC\frac{du_C}{dt}$$

$$\mathrm{E} = \mathrm{a} + be^{-\frac{t}{\tau}} - \mathrm{RC}.\frac{b}{\tau}e^{-\frac{t}{\tau}}$$

$$0 = -\mathrm{E} + \mathrm{a} + be^{-\frac{t}{\tau}} - \mathrm{RC}.\frac{b}{\tau}e^{-\frac{t}{\tau}}$$

$$0 = -\mathrm{E} + \mathrm{a} + be^{-\frac{t}{\tau}}\left[1 - \frac{RC}{\tau}\right]$$

Exercise 1:

$$0 = -E + a + be^{-\frac{t}{\tau}}\left[1 - \frac{RC}{\tau}\right]$$

$$-E + a = 0$$

$$a = E$$

$$1 - \frac{RC}{\tau} = 0$$

$$\frac{1}{1} = \frac{RC}{\tau}$$

$$\tau = RC$$

Exercise 1:

$$u_C = a + b.e^{-\frac{t}{\tau}}$$

$$u_C = E + b.e^{-\frac{t}{RC}}$$

At $t = 0;\ u_C = 0$

$$0 = E + b.e^{-\frac{0}{RC}}$$

$$0 = E + b.e^{0}$$

$$0 = E + b$$

$$b = -E$$

$$u_C = E - E.e^{-\frac{t}{RC}}$$

$$u_C = E(1 - e^{-\frac{t}{RC}})$$

Exercise 1:

4.Deduce that the expression of the current is: $i = \frac{E}{R}.e^{-\frac{t}{RC}}$.

$$u_C = E - E.e^{-\frac{t}{RC}}$$

$$i = C.\frac{du_C}{dt}$$

$$\frac{du_C}{dt} = \frac{E}{RC}.e^{-\frac{t}{RC}}$$

$$i = C.\left(\frac{E}{RC}.e^{-\frac{t}{RC}}\right)$$

$$i = \frac{E}{R}.e^{-\frac{t}{RC}}$$

Exercise 1:

5. The ammeter (A) indicates a value $I_0 = 5mA$ at $t_0 = 0$. Deduce the value of R.

$$i = \frac{E}{R} . e^{-\frac{t}{RC}}$$

At $t_0 = 0;\ I_0 = 5 \times 10^{-3} A$

$$I_0 = \frac{E}{R} . e^{-\frac{0}{RC}}$$

$$I_0 = \frac{E}{R}$$

$$R = \frac{E}{I_0}$$

$$R = \frac{10}{5 \times 10^{-3}}$$

$$R = 2000\Omega$$

Exercise 1:

6. Write the expression of $u_R = u_{DN}$ in terms of E, R, C and t.

7. At an instant $t = t_1$, the voltage across the capacitor is $u_C = u_R$.

a. Show that $t_1 = RC.\ln 2$.

b. Calculate the value of C knowing that $t_1 = 1.4ms$.

Exercise 1:

6. Write the expression of $u_R = u_{DN}$ in terms of E, R, C and t.

Using ohm's law of resistor

$$u_R = R.i$$

$$u_R = R.\frac{E}{R}.e^{-\frac{t}{RC}}$$

$$u_R = E.e^{-\frac{t}{RC}}$$

Exercise 1:

7. At an instant $t = t_1$, the voltage across the capacitor is $u_C = u_R$.

a. Show that $t_1 = RC.ln2$.

$$u_C = u_R$$

$$E - Ee^{-\frac{t_1}{RC}} = E.e^{-\frac{t_1}{RC}}$$

$$E = +E.e^{-\frac{t_1}{RC}} + E.e^{-\frac{t_1}{RC}}$$

$$E = 2E.e^{-\frac{t_1}{RC}}$$

$$1 = 2.e^{-\frac{t_1}{RC}}$$

$$Ln(1) = Ln(2.e^{-\frac{t_1}{RC}})$$

$$0 = Ln2 + Ln(e^{-\frac{t_1}{RC}})$$

$$0 = Ln2 - \frac{t_1}{RC}Ln(e)$$

$$0 = Ln2 - \frac{t_1}{RC}$$

$$t_1 = RC.Ln2$$

Exercise 1:

b. Calculate the value of C knowing that $t_1 = 1.4ms$

$$t_1 = RC.Ln2$$

$$C = \frac{t_1}{R.Ln2} \Rightarrow C = \frac{1.4 \times 10^{-3}}{2000.Ln2}$$

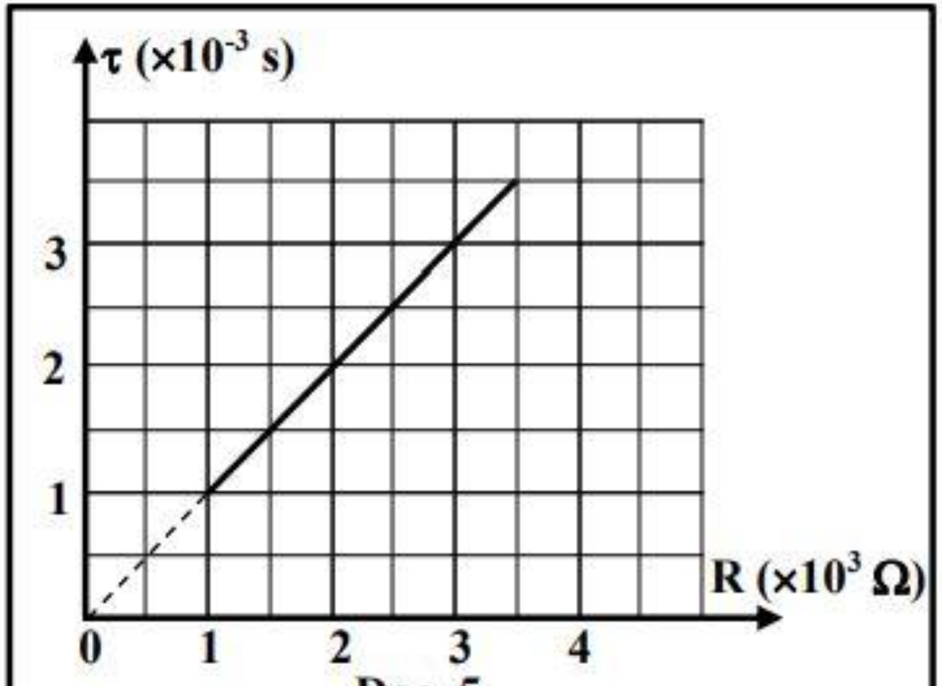
$$C = 1 \times 10^{-6} F$$

Exercise 1:

8.To verify the value of C, we vary the value of R. Document 5 represents τ as a function of R.

a.Show that the shape of the curve in document 5 agrees with the expression of τ obtained in part 3.

b.Using the curve of document 5, determine again the value of C.

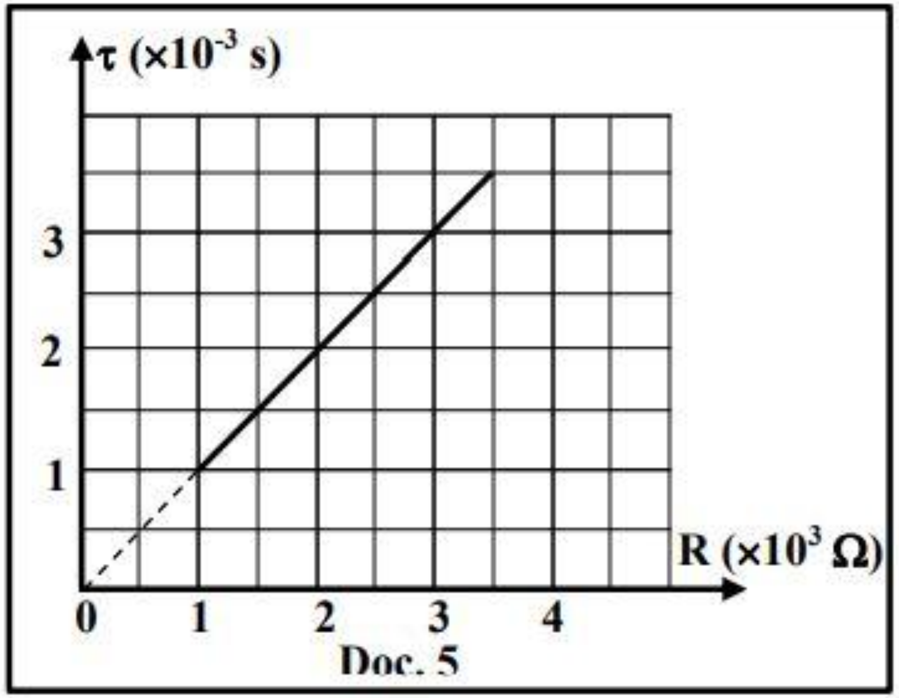


Doc. 5

Exercise 1:

a. Show that the shape of the curve in document 5 agrees with the expression of τ obtained in part.

The curve is a straight line passing through the origin with a positive slope, then it agrees with the expression $\tau = RC$.



Doc. 5

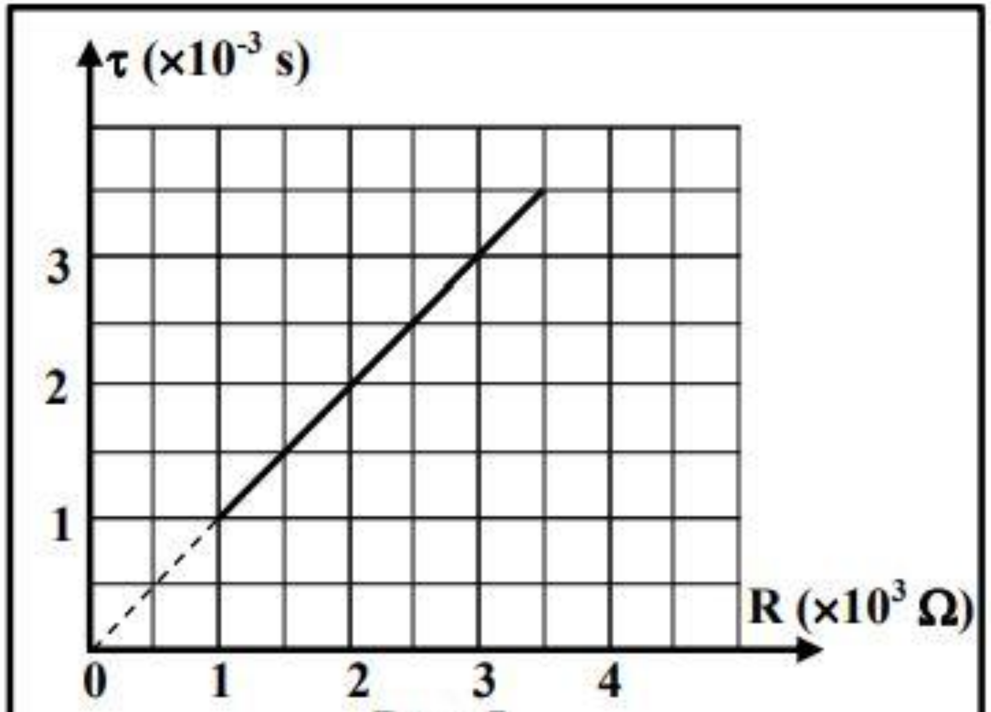
Exercise 1:

b. Using the curve of document 5, determine again the value of C

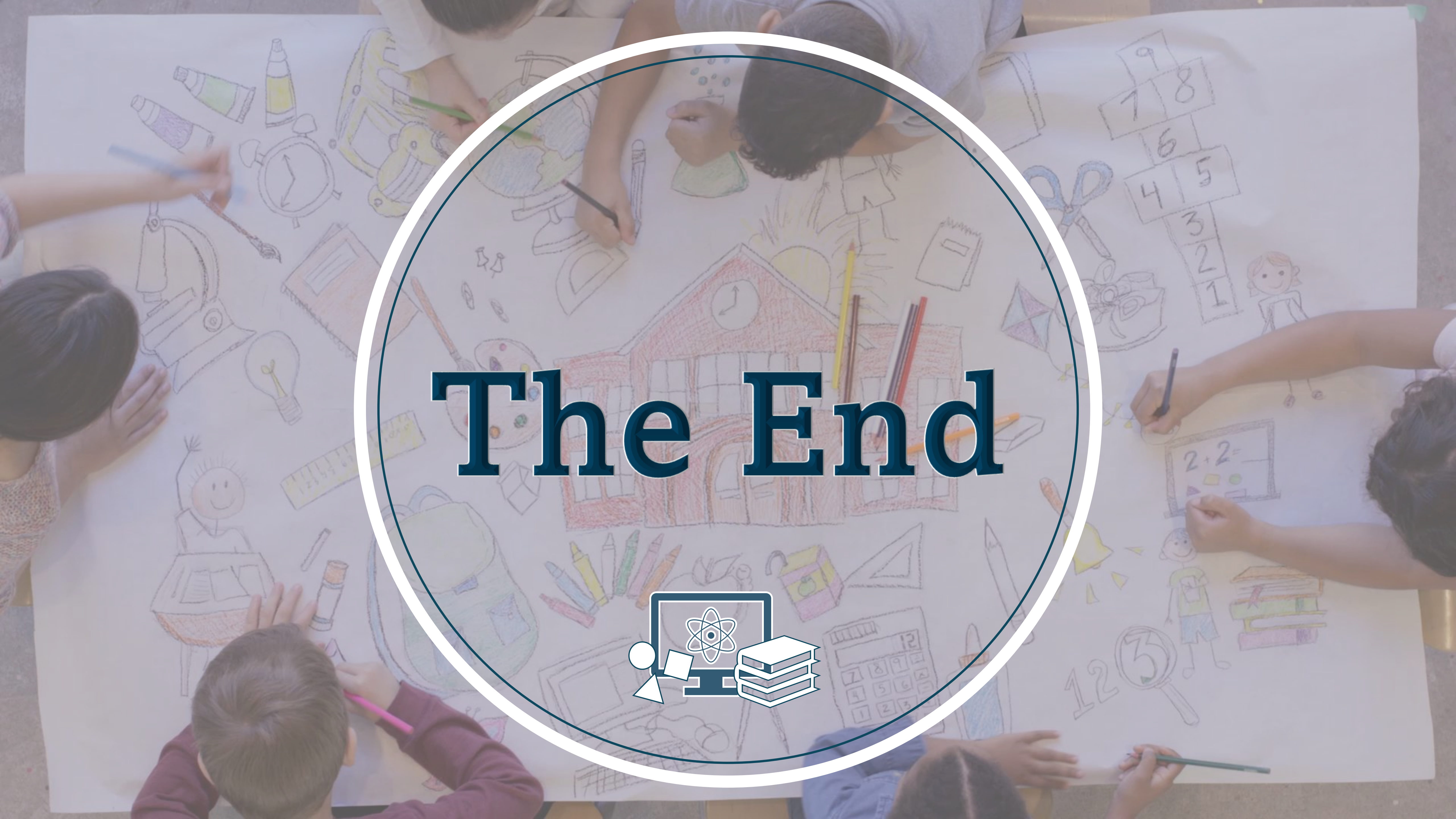
$$Slope = C = \frac{\tau_2 - \tau_1}{R_2 - R_1}$$

$$C = \frac{(2-1) \times 10^{-3}}{(2-1)10^3}$$

$$C = 2000\Omega$$



Doc. 5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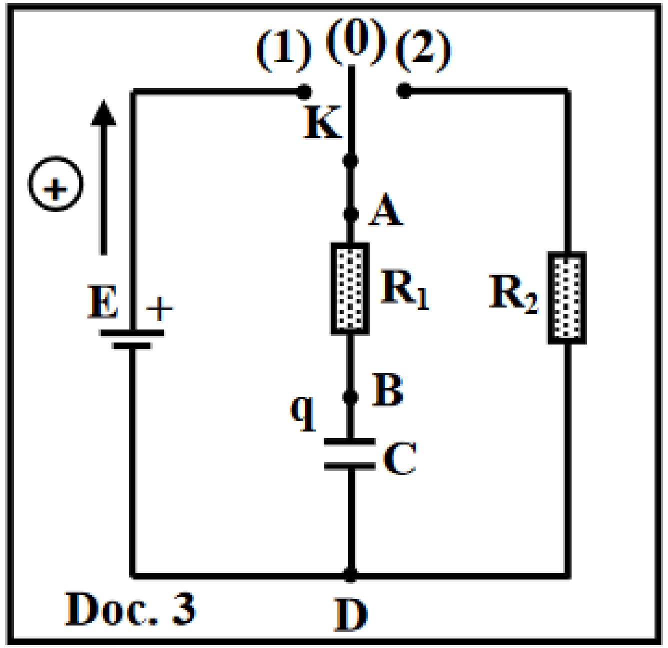
Think then Solve

Exercise 2:

We set up the circuit of document 3 that includes:

- **an ideal battery of electromotive force E = 10V.**
- **two resistors of resistances $R_1 = R_2 = 4K\Omega$.**
- **a capacitor of capacitance C and a switch K.**



Charging the capacitor:

The switch K is initially at position (0) and the capacitor is uncharged.

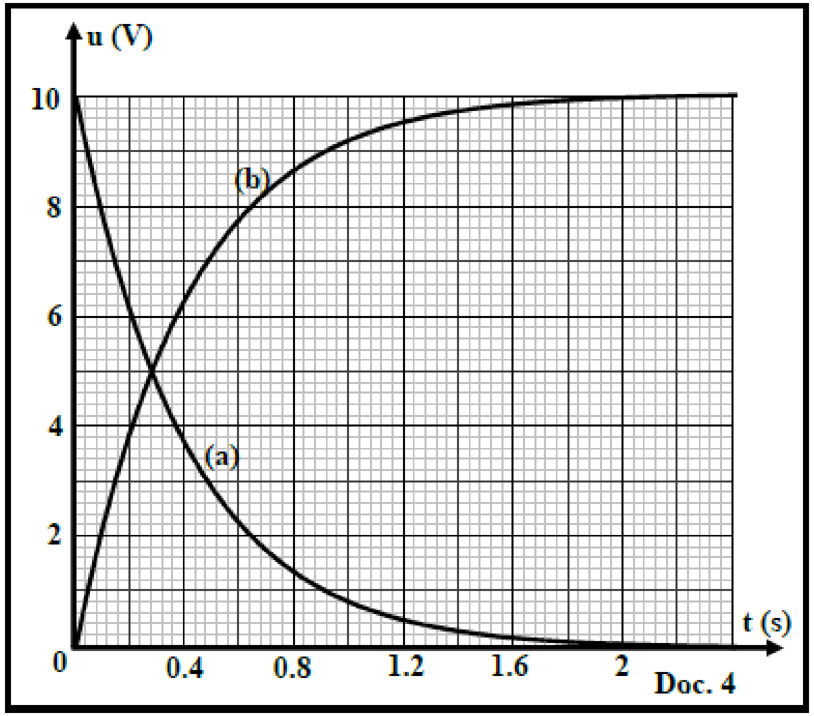
At the instant $t_0 = 0$, K is turned to position (1) and the charging process of the capacitor starts.

At an instant t, plate B of the capacitor carries a charge q and the circuit carries a current i.

Exercise 2:

An appropriate device allows us to display the voltage $u_{AB} = u_{R1}$ across the resistor and the voltage $u_{BD} = u_C$ across the capacitor.

Curves (a) and (b) of document 4 show these voltages as functions of time.



1.1) Curve (a) represents u_{R1} and curve (b) represents u_C. Justify

1.2) The time constant of this circuit is given by $\tau_1 = R_1 C$

1.2.1) Using document 4, determine the value of τ_1.

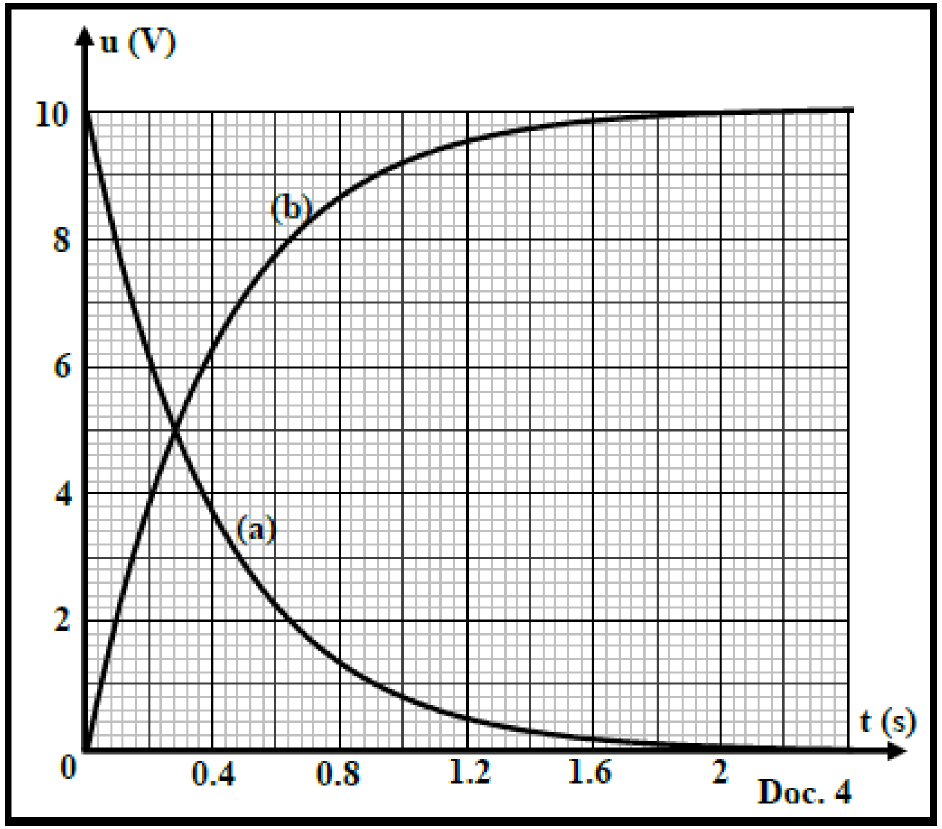
Exercise 2:

1.2.2) Deduce the value of C.

1.3) Calculate the time «t_1» needed by the capacitor to practically become completely charged

Exercise 2:

1.1) Curve (a) represents u_{R1} and curve (b) represents u_C. Justify.



Curve (a): $u_{R1} = R_1 i$ is directly proportional to the current in the circuit.

During the charging process the current decreases so u_{R1} decreases.

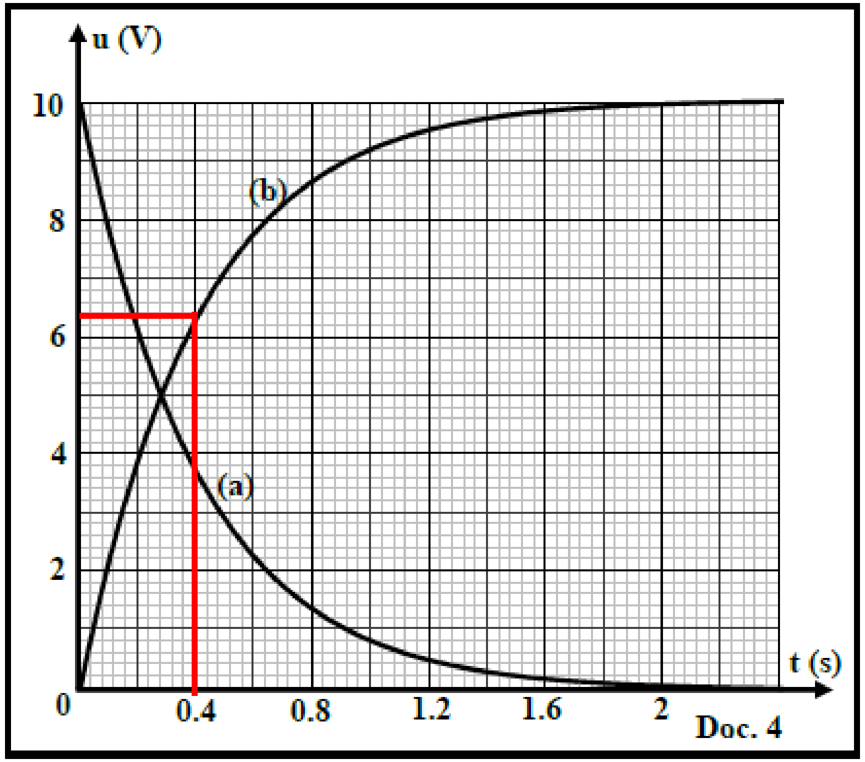
Curve (b): $u_{BD} = u_C$, During charging process q increases so u_C increases.

Exercise 2:

E=10V, $R_1 = R_2 = 4K\Omega$

1.2) The time constant of this circuit is given is $\tau_1 = R_1 C$

1.2.1) Using document 4, determine the value of τ_1.



At $t = \tau_1$:

$$u_C = 0.63 \times E$$

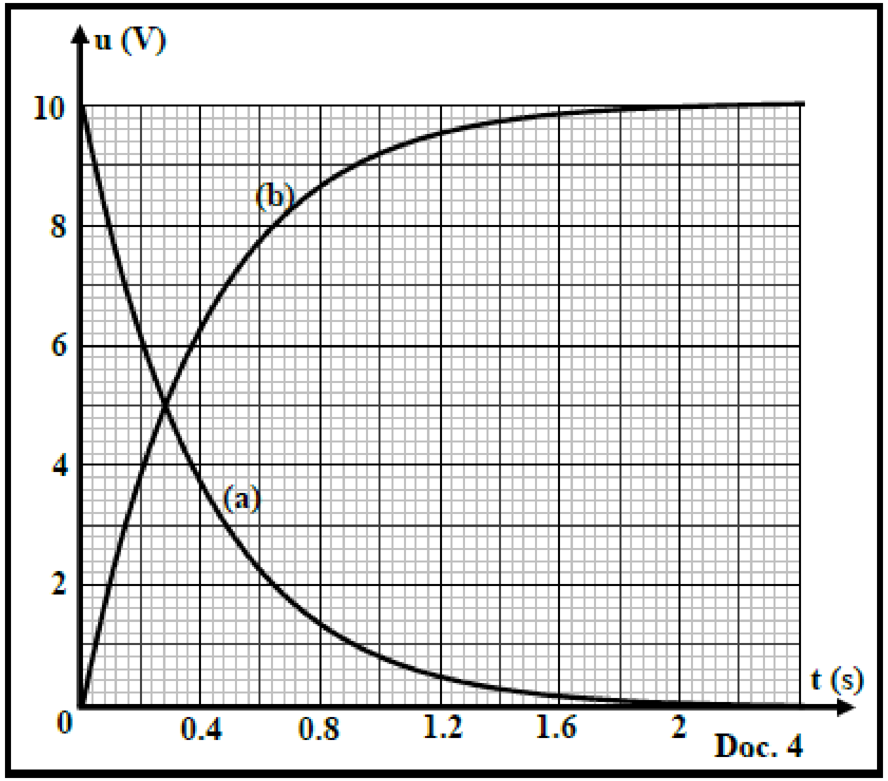
$$u_C = 0.63 \times 10 = 6.3V$$

Then: $\tau_1 = 0.4sec$

Exercise 2:

E=10V, $R_1 = R_2 = 4K\Omega$

1.2.2) Deduce the value of C.



$$\tau_1 = R_1 C \Rightarrow C = \frac{\tau_1}{R_1}$$

$$C = \frac{0.4}{4000} \Rightarrow C = 1 \times 10^{-4} F$$

1.3) Calculate the time «t_1» needed by the capacitor to practically become completely charged.

The capacitor to practically become completely charged at $t_1 = 5\tau_1$

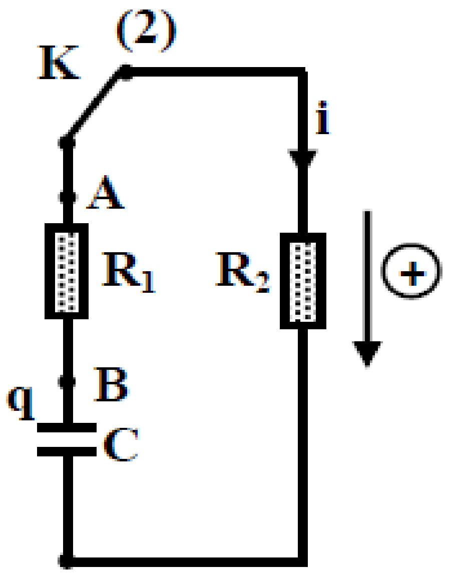
$$t_1 = 5 \times 0.4 \Rightarrow t_1 = 2s$$

Exercise 2:

2) Discharging the capacitor:

The capacitor is completely charged. At an instant taken as a new initial time $t_0 = 0$, the switch K is turned to position (2), and the capacitor starts discharging through the resistors of resistances R_1 and R_2.

At an instant t the circuit carries a current i (Doc. 5).



Doc. 5

2.1) Show, using the law of addition of voltages, that the differential equation which governs u_C is:

$$RC\frac{du_C}{dt} + u_C = 0 \text{ where } R = R_1 + R_2$$

Exercise 2:

2.2) The solution of this differential equation is of the form:

$u_C = Ee^{\frac{-t}{\tau_2}}$, **where** τ_2 **is the time constant of the circuit of document 5. Determine the expression of** τ_2 **in terms of R and C.**

2.3) Verify that the time needed by the capacitor to practically become completely discharged is $t_2 = 5\tau_2$**.**

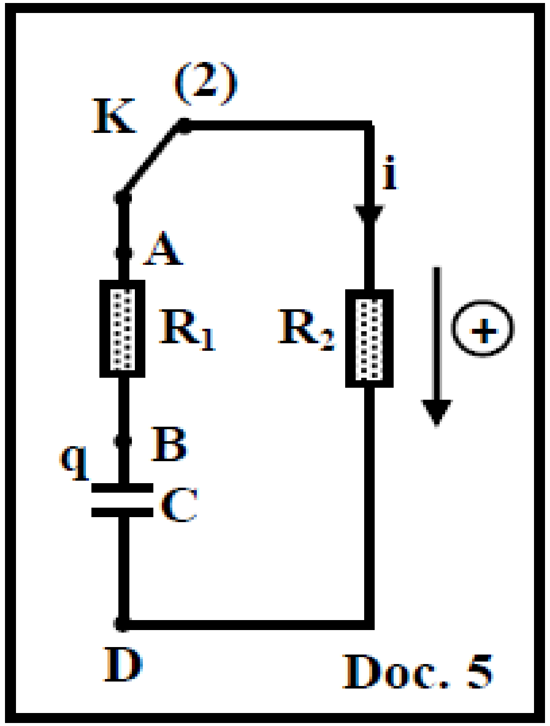
3) Duration of charging and discharging the capacitor

Show, without calculation, that « t_2**» is greater than «**t_1**».**

Exercise 2:

2.1) Show, using the law of addition of voltages, that the differential equation which governs u_C is:

$$RC\frac{du_C}{dt} + u_C = 0 \text{ where } R = R_1 + R_2$$



$$u_C = u_{R1} + u_{R2} \Rightarrow u_C = R_1 i + R_2 i$$

$$u_C = (R_1 + R_2)i \Rightarrow u_C - (R_1 + R_2)i = 0$$

$$i = -\frac{dq}{dt} \text{ and } q = Cu_C \Rightarrow i = -C\frac{dU_C}{dt}$$

$$u_C + (R_1 + R_2)C\frac{du_C}{dt} = 0$$

Exercise 2:

2.2) The solution of the differential equation is $u_C = Ee^{\frac{-t}{\tau_2}}$, where τ_2 is the time constant of the circuit.

Determine the expression of τ_2 in terms of R and C.

$$u_C = Ee^{\frac{-t}{\tau_2}} \Rightarrow \frac{du_C}{dt} = -\frac{E}{\tau_2}e^{\frac{-t}{\tau_2}}$$

Subsitute u_C and $\frac{du_C}{dt}$ in differential equation

$$u_C + RC.\frac{du_C}{dt} = 0$$

$$E.e^{\frac{-t}{\tau_2}} - RC\frac{E}{\tau_2}e^{\frac{-t}{\tau_2}} = 0$$

$$E.e^{\frac{-t}{\tau_2}}\left[1 - \frac{RC}{\tau_2}\right] = 0$$

$$1 - \frac{RC}{\tau_2} = 0 \qquad \tau_2 = RC$$

Exercise 2:

2.3) Verify that the time needed by the capacitor to practically become completely discharged is $t_2 = 5\tau_2$.

$$u_C = Ee^{\frac{-t}{\tau_2}}$$

At $t_2 = 5\tau_2$:

$$u_C = Ee^{\frac{-5\tau_2}{\tau_2}}$$

$$u_C = Ee^{-5} \cong 0$$

Since at t_2, the voltage across the capacitor is zero, then; the capacitor is practically completely discharged

Exercise 2:

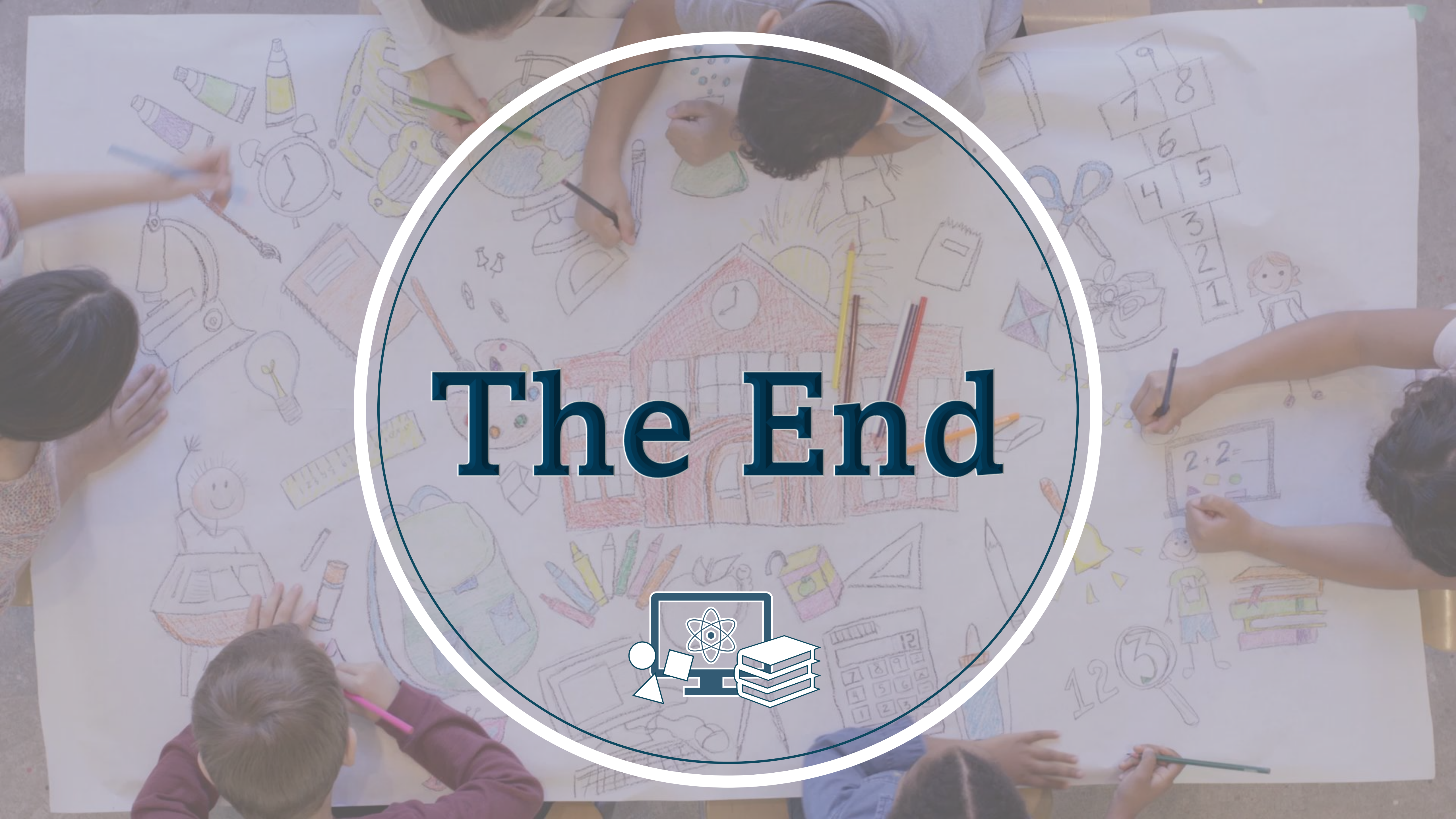
3) Duration of charging and discharging the capacitor

Show, without calculation, that « t_2» is greater than «t_1».

$$t_2 = 5(R_1 + R_2)C$$

$$t_1 = 5R_1C$$

Since $(R_1 + R_2) > R_1$ then $t_2 > t_1$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